Localization in Plane Strain Compression of Fluid-Saturated Rock

Roman Y. Makhnenko and Joseph F. Labuz

Abstract Localization in terms of a bifurcation from a homogeneous pattern of deformation is predicted to be different if the boundary conditions are drained or undrained. This phenomenon is evaluated in plane strain compression experiments with water-saturated Berea sandstone, where corresponding poroelastic and inelastic properties were measured. Non-uniformity of lateral deformation and clustering of acoustic emission events were used as the experimental evidence of localization, which occurred soon after the peak stress in both drained and undrained cases. These observations are in reasonable agreement with an analysis that predicted negative values of the critical hardening modulus (localization post-peak) for fluid-saturated sandstone under plane strain loading conditions.

1 Introduction

Elastic, inelastic, and strength characteristics of fluid-saturated rock depend on the rate of deformation and boundary conditions. Moreover, studies show that the conditions for localization of deformation can also be different for specimens tested in drained and undrained regimes Rudnicki and Rice [6], Rudnicki [5]. Even though localization may result from inhomogeneities or stress concentrations, an alternative point of view is that this phenomenon is a bifurcation from a smoothly varying pattern of deformation, where the constitutive description of homogeneous deformation can admit a solution that is compatible with boundary conditions for further homogeneous deformation, but corresponding to non-uniform deformation in a planar zone Rudnicki and Rice [6]. In the following, this approach is compared with

---

R.Y. Makhnenko (✉)
Department of Civil and Environmental Engineering,
University of Illinois at Urbana-Champaign, Champaign, IL, USA
e-mail: romanmax@illinois.edu

J.F. Labuz
Department of Civil, Environmental, and Geo-Engineering,
University of Minnesota – Twin Cities, Minneapolis, MN, USA

E. Papamichos et al. (eds.), *Bifurcation and Degradation of Geomaterials with Engineering Applications*, Springer Series in Geomechanics and Geoengineering, DOI 10.1007/978-3-319-56397-8_5

two experimental methods of detecting the onset of localization: one is based on the deviation of uniform deformation of the specimen and the other is based on clustering of acoustic emission (AE) events. Experiments were performed on water-saturated Berea sandstone to determine its poroelastic and inelastic properties and to observe localization in drained and undrained plane strain compression.

2 Background

Rudnicki and Rice [6] suggested a constitutive description that is compatible with boundary conditions for homogeneous deformation but non-uniform deformation in a planar zone. The specific constitutive relations for (confined and compressed) brittle rock can be written in terms of a hardening modulus h. For the case of changing mean stress P (P' = Terzaghi's effective mean stress), h is related to the tangent modulus $h_{tan} = (d\tau - \mu dP')/(d\gamma - d\tau/G)$, where γ = shear strain, τ = shear stress, μ = friction coefficient, and G = shear modulus. The RR analysis predicts a critical value of the hardening modulus at which localization occurs:

$$h_{crit} = \frac{G(1+\nu)}{9}\left[\frac{(\beta-\mu)^2}{(1-\nu)} - \frac{1}{2}\left(\sqrt{3}\eta+\beta+\mu\right)^2\right] \tag{1}$$

Here β is the dilatancy factor and $\eta = \sqrt{3}\cdot s_2/\tau$ with s_2 being the intermediate principal value of the deviator stress tensor. This criterion is valid for both dry and drained deformation (constant pore pressure). The localization of deformation happens when the condition $h = h_{crit}$ is met (bifurcation point).

If the drained response is rate-independent, then the alternative limit of undrained deformation is also rate-independent and a similar bifurcation analysis can be applied. Rudnicki [5] proposed a form of the constitutive relation for undrained response, which is obtained from the drained response (Eq. 1) by substitutions that involve Skempton's B coefficient and undrained Poisson's ratio ν_u. The following assumptions are involved: (i) the elastic portion of strain increments can be described by linear, isotropic poroelasticity; (ii) the role of the pore pressure p in the inelastic strain increments is included by replacing the mean stress P by Terzaghi's effective mean stress P'; (iii) the inelastic increment in the apparent porosity is equal to the inelastic volume strain increment. From Rudnicki [5], the critical undrained hardening modulus for localization H_{crit} can be written as:

$$H_{crit} = \frac{G(1+\nu_u)}{9}\left[\frac{(1-B)^2(\beta-\mu)^2}{(1-\nu_u)} - \frac{1}{2}\left(\sqrt{3}\eta+(1-B)(\beta+\mu)\right)^2\right] \tag{2}$$

3 Methods

A Vardoulakis and Goldscheider [7] type plane strain apparatus was used for this study and modified for fluid-saturated rock testing of 100 × 87 × 44 mm specimens Makhnenko and Labuz [2]. The apparatus combines the positive features of a constitutive (plane strain) compression test, such that the two-dimensional material behavior can be evaluated, and a direct shear test, such that the characteristics of the shear band can be measured. Lateral displacement is measured at two positions, which allows an evaluation of non-uniform deformation and the onset of localization. Additionally, eight AE sensors are used to monitor microcracking (Fig. 1). AE locations can be determined to evaluate the onset of localization. For example, a criterion based on the value of the fractal dimension of the events positions has been proposed Iverson et al. [1]. However, a simple criterion is used in this study: if 90% of continuously located AE events with accuracy $r_a = 3$ mm are situated within sphere of $3r_a$ diameter (=9 mm or 3 standard deviations), the deformation is described to be localized (Fig. 1b).

Another method of determining localization is to detect when the deformation of the specimen is no longer homogeneous. Two LVDTs are positioned to measure the lateral displacement of the specimen (Fig. 1a). Typically, the output of the two LVDTs is changing at a uniform rate up to the peak axial stress, and just after the peak, one is approximately constant and the other continues to increase (Fig. 2a). This behavior is related to the formation of the shear band, when one part of the specimen slides and the other part remains stationary. The point where the lateral displacements change rate is taken as the onset of localization.

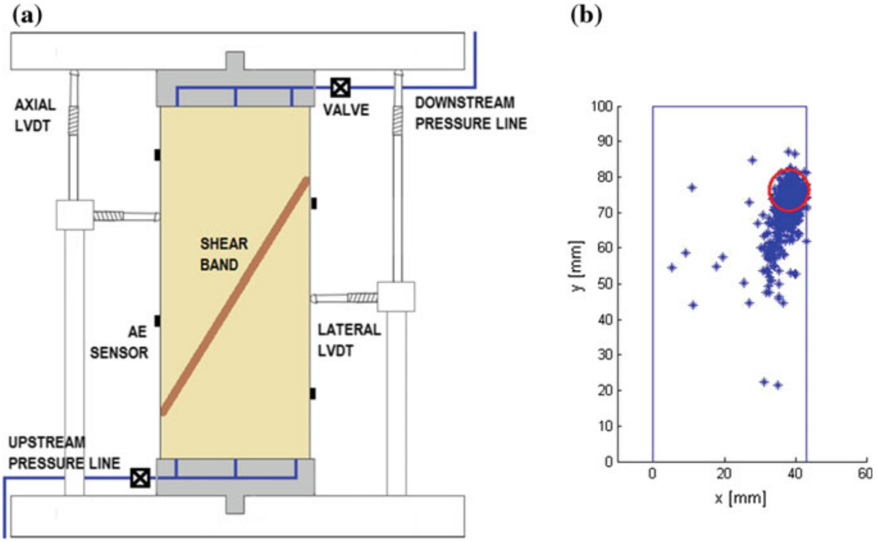
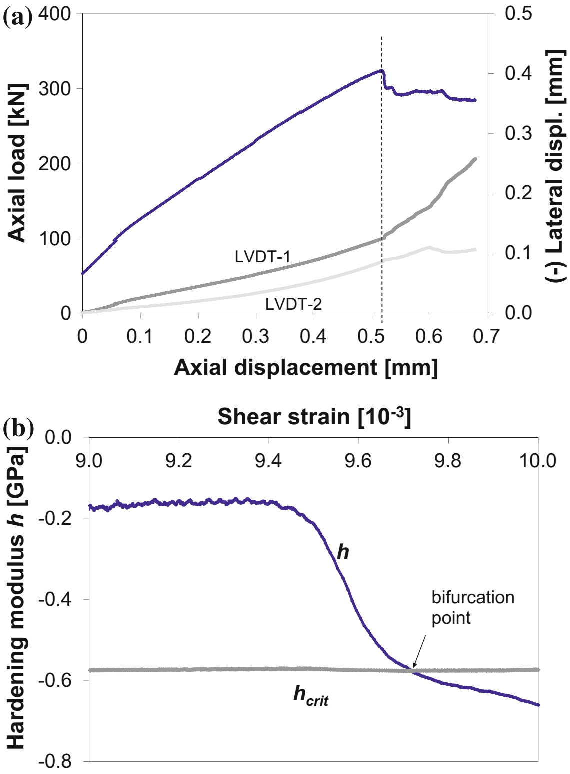


Fig. 1 **a** Sketch of the experimental setup and **b** AE events clustering in the sandstone

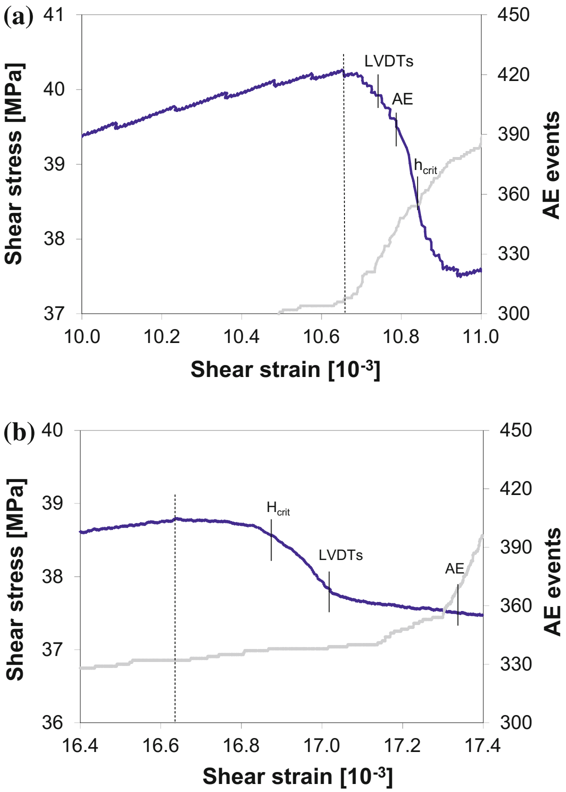
Fig. 2 Bifurcation point predicted by **a** onset of non-homogeneous deformation (*dashed line*) and **b** critical hardening modulus method for dry plane strain experiment



4 Results

Specimens of Berea sandstone were fully saturated with water and loaded to failure under plane strain conditions. Three drained and three undrained compression experiments were performed and exhibited similar results in terms of detecting the onset of localization, so the data here is reported only for one drained and one undrained test. The drained test was performed with constant (i) pore pressure = 5 MPa and (ii) minimum principal stress σ_3 = 10 MPa. Precise measurements of axial and lateral stresses and strains allowed for calculation of the material parameters: G = 4.6 GPa, μ = 0.79, and β = 0.41, where friction μ and dilatancy β are reported at the peak Makhnenko and Labuz [3, 4]. This provided calculation of h_{crit} from Eq. (1). The undrained plane strain compression test was performed at σ_3 = const = 10 MPa and the initial value of pore pressure was 3 MPa. Pore pressure increased to 7 MPa during elastic compaction and then decreased to 4.5 MPa when the specimen was dilating before the peak. Values of B = 0.61 and ν_u = 0.35 Makhnenko and Labuz [4] were used to calculate H_{crit} for the undrained plane strain compression test.

Fig. 3 Localization points determined by three methods: non-uniform lateral deformation (LVDTs), $h = h_{crit}$, and clustering of acoustic emission (AE) for **a** drained and **b** undrained plane strain compression tests. *Vertical dotted line* designates the shear strain at peak stress



Additionally, onsets of non-uniform rock deformation and AE clustering were obtained from the proposed methods. The results of the three localization criteria for drained and undrained plane strain compression experiments are presented in Fig. 3. The response of the undrained specimen is significantly less brittle, which is explained by the increase in effective minimum principal stress σ_3' caused by dilatant pore pressure decrease around and after axial peak stress Makhnenko and Labuz [3].

5 Discussion and Conclusions

The onset of localization in water-saturated Berea sandstone was determined by three methods: non-uniformity of specimen deformation detected by lateral LVDTs, clustering of AE events, and a critical value of the hardening modulus computed from measured elastic and inelastic material properties. In general, for both drained and undrained compression tests, all three methods provided localization happening soon after the peak axial stress was reached. In the case of drained compression, all the prediction methods for the onset of localization are consistent (lie within 0.15×10^{-3} shear strain) whereas in the undrained test AE clustering was delayed by 0.5×10^{-3} shear strain and the AE rate is significantly lower. It should be noted that the detection of the onset of localization with AE locations is sensitive to the chosen criterion. Also, stress and directional dependency of some material parameters (e.g. B, ν, and G), as well as their degradation during material damage, are not considered here, but influence the reported values of the critical hardening modulus.

Some discrepancy in the results of observed (from non-uniformity of deformation and AE) and predicted (from the critical value of the hardening modulus) localization also could be explained by the use of a relatively low effective minimum principal stress (below 10 MPa), where for the drained test, the failure plane was quite steep, and for the undrained case, the failure mode exhibited features associated with axial splitting.

References

1. Iverson, N., Kao, C.-S., Labuz, J.F.: Clustering analysis of AE in rock. J Acoustic Emission **25**, 364–372 (2007)
2. Makhnenko, R., Labuz, J.: Plane strain testing with passive restraint. Rock Mech. Rock Eng. **47**(6), 2021–2029 (2014)
3. Makhnenko, R.Y., Labuz, J.F.: Dilatant hardening of fluid-saturated sandstone. J Geophys Res Solid Earth **120**, 909–922 (2015)
4. Makhnenko, R.Y., Labuz, J.F.: Elastic and inelastic deformation of fluid-saturated rock. Phil Trans R Soc A **374**, 20150422 (2016)
5. Rudnicki, J.W.: Localization in undrained deformation. In: Proceedings of the 4th Biot Conference on Poromechanics, Columbia University, New York (2009)
6. Rudnicki, J.W., Rice, J.R.: Conditions for the localization of deformation in pressure-sensitive dilatant materials. J Mech Phys Solids **23**, 371–394 (1975)
7. Vardoulakis, I., Goldscheider, M.: Biaxial apparatus for testing shear bands in soils. In: Proceedings of 10th International Conference on Soil Mechanics and Foundation Engineering, pp. 819–824. Balkema, Rotterdam (1981)