On the Unjacketed Moduli of Sedimentary Rock

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Abstract

Experimental techniques have been developed to measure two poroelastic parameters: unjacketed bulk modulus K_s' and unjacketed pore modulus K_s''. K_s' is measured on samples with no jacket that are instrumented with strain gages, confining fluid (oil) penetrates inside the material and hydrostatic mean stress is equal to pore pressure, so the overal unjacketed bulk modulus can be calculated. K_s'' is measured on water-saturated rock by determining the change in pore volume of the specimen from accurate measurements on pore pressure controller, while the difference between mean stress and pore pressure is preserved constant. K_s' and K_s'' are measured to be pressure-independent for Berea sandstone and Opalinus clay (shale). $K_s' \approx K_s''$ for the clay-rich shale, but K_s'' is smaller than K_s' and bulk modulus of quartz for quartz-rich Berea sandstone. This result is explained by the presence of non-connected porosity.

INTRODUCTION

In poroelastic analyses, it is important to consider the compressibility of the solid constituents forming the rock, and often times solid bulk modulus K_s is assumed to be the same as the dominant mineral bulk modulus. In general, there are two different parameters describing solid compressibility: the unjacketed bulk modulus K_s' and the unjacketed pore modulus K_s'' associated with constant Terzaghi effective stress testing. K_s' is related to the total volume change in unjacketed loading, while K_s'' is associated with the change only in the pore volume (Rice and Cleary 1976). If a rock has fully connected pore space, all points of the solid phase may be taken as elastically isotropic with the same local bulk modulus K_s, and both fluid and solid are chemically inert for the time scale of the tests, it can be shown that $K_s = K_s' = K_s''$. However, this assumption may not be valid for sedimentary rock where intergranular cement and pore lining have different elastic properties and non-connected pore space is present (Detournay and Cheng 1993).

Technical difficulties in measuring K_s' and especially K_s'' directly often lead to improper assumptions of their values and hence influence model predictions for reservoir behavior. This paper presents experimental techniques that were developed to measure the unjacketed bulk moduli and discussion on the material properties that affect these parameters.

BACKGROUND

Classical constitutive relationship for fluid-filled solid that is experiencing linear elastic or viscoelastic deformation can be written without taking into account the individual contributions of solid and fluid constituents (Biot 1941, Detournay and Cheng 1993, Yarushina and Podladchikov 2015). These contributions can be considered if a micromechanical approach is used, which is conveniently introduced through the so-called unjacketed boundary condition. The unjacketed test was proposed by Biot and Willis (1957) and it is characterized by equal increments in mean stress P and pore pressure p, $\Delta P = \Delta p$: the specimen with no jacket or membrane is loaded by a fluid that is allowed to penetrate inside the rock and thus equilibrate the mean stress with the pore pressure. The test can also be carried out on a jacketed specimen by imposing equal increments of pore pressure and mean stress, without the requirement of the equality of P and p. Rice and Cleary (1976) introduced two micromechanical material constants: unjacketed bulk modulus K_s' and unjacketed pore modulus K_s''

$$K_s' = V \left.\frac{\Delta p}{\Delta V}\right|_{\Delta p=\Delta P} \tag{1}$$

$$K_s'' = V_p \left.\frac{\Delta p}{\Delta V_p}\right|_{\Delta p=\Delta P} \tag{2}$$

Generally, K_s' and K_s'' are different; K_s' is related to the change in the total volume V in unjacketed compression, while K_s'' is associated with the change only in the pore volume V_p. However, under certain conditions, they can be both identified with the bulk modulus of the solid constituent $K_s = V_s\,(\Delta p/\Delta V_s)\big|_{\Delta p=\Delta P}$, V_s = the solid volume. Consider an ideal porous rock characterized by a fully connected pore space with a microscopically homogeneous and isotropic matrix. Also, assume the bulk moduli of the major mineral constituents do not differ by a large amount and both fluid and solid are chemically inert for the time scale of the tests. In such a case, when the rock is subjected to a uniform pressure $\Delta P = \Delta p$ everywhere in the solid constituent, it deforms as if all pores were filled with solid material (Geertsma 1957, Nur and Byerlee 1971). Therefore, the solid component and the rock framework experience a uniform volumetric strain without any shape change

$$\frac{\Delta V}{V} = \frac{\Delta V_s}{V_s} = \frac{\Delta V_p}{V_p} \tag{3}$$

This suggests no change in porosity under this type of loading for an ideal porous material described. Rice and Cleary (1976) showed the following identity between the two solid moduli:

$$K_s = K_s' = K_s'' \tag{4}$$

However, this identity may not be valid for a sedimentary rock that does not fully represent an ideal porous material. Existence of inhomogeneities such as non-connected pore space (Detournay and Cheng 1993, Makhnenko and Labuz 2016) as well as intergranular cement and pore lining that have different elastic properties (Green and Wang 1986, Hart and Wang 2010) could cause a potential difference between K_s' and K_s''.

METHODS

Different approaches to measuring K_s' for porous rock are discussed in Makhnenko and Labuz (2013). It was concluded that hydrostatic compression on prismatic specimens (approx. 50 x 35 x 35 mm) instrumented with sets of strain rosettes, placed inside a pressure chamber, and subjected to mean stress P provide the most accurate and consistent results. Volume strain ε can be calculated as the trace of the three-dimensional strain tensor ε_{ij} (the first invariant), and hence the measurement of the strains in any three perpendicular directions are enough for determining ε. The confining fluid (oil) is allowed to penetrate into the rock and is used because it has no electrical effect on strain gage connections and no chemical effect on the tested rock in the short term. After at least 24 hours of saturation at 20 – 30 MPa pore pressure, unjacketed loading ($P = p$ and $\Delta P = \Delta p$) up to 60 MPa and unloading is performed. The unjacketed bulk modulus is then calculated from equation (1).

Another geometry that allows direct measurements of applied principal stresses and corresponding strains is a cylindrical specimen tested in conventional triaxial (axisymmetric) compression (H = 104 mm, D = 50.8 mm). To find the unjacketed pore volume modulus K_s'', accurate determination of pore volume changes is required. The variation of the pore volume evaluated from the amount of fluid exchanged between the rock and the pressure/volume controller when applying equal increments of mean stress and pore pressure could, in principle, give K_s''. The experiment needed to measure K_s'' is performed in two steps at constant temperature. The first step involves the increase of the external pressure P by ΔP while pore pressure p is held constant. From this step, the only component of the total volume change (recorded by the syringe pump) that needs correction is the volume change of a portion of the tubing inside the pressure vessel. The second step involves the increase in pore pressure p by $\Delta p = \Delta P$ while external pressure is held constant. In this step, since the fluid pressure changes, detailed corrections are needed. The fluid volume change associated with the compression or expansion of the interstitial fluid needs to be considered

$$\Delta V_f = V_f \frac{\Delta p}{K_f} \tag{5}$$

where V_f is the volume of the fluid occupying the pore space. K_f is the bulk modulus of fluid, which is measured to be 2.2 GPa for water at a few MPa pressure and room temperature. The remaining volume changes recorded by the syringe pump are due to the system and the pore volume of the rock. The system volume changes during the second step consist of the syringe pump piston, valves, and tubes inside and outside the vessel, as well as the pressure transducer.

To find these system responses corresponding to the two steps mentioned, it is necessary to run calibration tests with a hollow cylindrical specimen in which the pore volume changes can be calculated from the elasticity solution for thick-walled cylinders (Zimmerman *et al*. 1985). Precise calibration tests using a hollow aluminum cylinder (E = 72.0 GPa, v = 0.34) were performed to find these system responses (Tarokh 2016).

RESULTS

Unjacketed bulk modulus K_s' is measured for Berea sandstone (85% quartz content) and Opalinus clay (Jurassic shale) with about 50-60% of clay minerals. Initially dry sandstone specimens are saturated at 60 MPa hydrostatic (oil) pressure and unloaded gradually within a few hours. The obtained K_s' = 30 GPa for Berea sandstone is smaller than the measured bulk modulus of quartz (37.0 GPa). The unjacketed pore modulus for the sandstone was measured directly by applying equal increments of the total (mean) stress and pore pressure on the specimens with initial P = 15 MPa and P = 30 MPa and p = 10 MPa. K_s'' = 22 GPa was found to be pressure-independent for Berea sandstone and $K_s'' < K_s'$ (Figure 1).

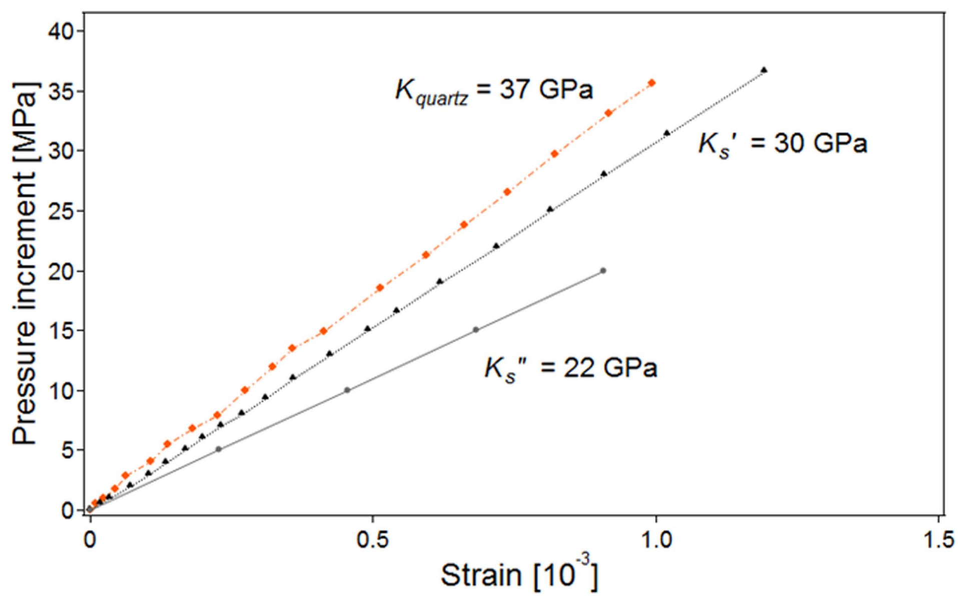

Figure 1. Unjacketed bulk modulus K_s' and unjacketed pore modulus K_s'' for Berea sandstone and bulk (solid) modulus of fused quartz.

The difference between K_s' and bulk modulus of quartz is explained by the presence of non-connected pore space in Berea sandstone. The microstructural analysis of thin sections of the sandstone vacuum-impregnated with epoxy stained fluorescent dye showed that some pores were not filled by the epoxy and hence contributed to the non-connected porosity. Precise measurements of solid (ρ = 2740 kg/m^3) and bulk density (ρ = 2060 kg/m^3) of the rock revealed that the total porosity of the Berea sandstone is equal to ϕ_{tot} = 0.25, which is larger than the interconnected porosity ϕ_{int} = 0.23 measured consistently by vacuum saturation and mercury intrusion porosimetry methods.

Direct measurements of unjacketed moduli for shales are rarely reported. Often times, K_s' is calculated from other poroelastic parameters (Makhnenko *et al.* 2011, Suarez-Rivera and Fjaer 2013) or estimated from mineral composition of shale (*e.g.*, Keller 2017). Tested Opalinus clay specimen has initial brine saturation of 0.85, it is fully saturated within ten days after gradually increasing oil pressure in the chamber to 60 MPa, and unloaded within 15 days to obtain K_s' = 9 GPa (Figure 2). Oil is a non-wetting fluid for the brine-saturated shale and is believed to occupy the rest of the pore space in the material at high pore pressure, which is confirmed by linearity of unjacketed material response. Low-pressure controller was utilized for measurements of unjacketed pore modulus in shale. Back pressure in the material was gradually increased to 2 MPa for 90 days to represent the *in-situ* brine pressure (K_{brine} = 2.0 GPa) at the Mont Terri Underground Rock Laboratory. Full saturation of the material was assured by measuring constant Skempton's B coefficient values (B = 0.93) while the effective mean stress was preserved to be the same, $P' = P - p$ = 0.5 MPa. K_s'' = 7 ± 2 GPa is reported for pore pressure range of p = 1.5 – 2.5 MPa and does not differ significantly from K_s' for Opalinus clay.

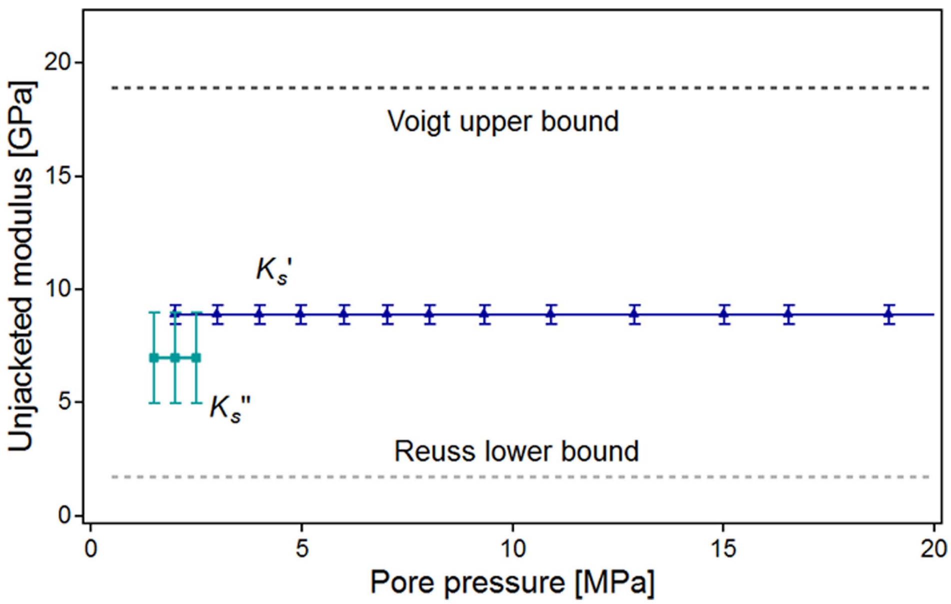

Figure 2. Measured unjacketed moduli: K_s' and K_s'' for Opalinus clay (shale) with calculated Voigt and Reuss boundaries for solid bulk modulus K_s.

Measurements of solid (ρ = 2750 kg/m^3) and bulk density (ρ = 2390 kg/m^3) of the shale provided ϕ_{tot} = 0.13, which is close to the interconnected porosity values ϕ_{int} = 0.12 – 0.13 that were obtained by using high pressure (400 MPa) mercury intrusion porosimetry technique and had some natural variation from one specimen to another. Voigt and Reuss bounds (Mavko *et al.* 2009) were estimated for the anisotropic shale based on the bulk moduli of minerals forming it: 50-60% clay, 15-25% calcite, 10-15% quartz, and 10% of highly-compressible biodegraded material (Bossart 2012). The order of magnitude difference between the upper and lower bounds demonstrates the importance of direct measurements of K_s' and K_s'' for multi-mineral rock. Measured material properties are summarized in Table 1.

Table 1. Summary of material properties: ϕ_{tot} – total porosity, ϕ_{int} – interconnected porosity, and K_s' and K_s'' – measured unjacketed moduli.

Material	ϕ_{tot}	ϕ_{int}	K_s' $[GPa]$	K_s'' $[GPa]$
Berea sandstone	0.25	0.23	30	22 ± 0.7
Opalinus clay (shale)	0.13	0.12-0.13	9	7 ± 2

Precision of K_s' measurements is within 2% and is determined from the accuracy of strain and hydrostatic pressure measurements under the assumption of full saturation. Evaluation of the precision of the reported direct measurement of K_s'' requires calculation of the accuracy of three measured parameters: V_p, $\Delta p = \Delta P$, and ΔV_p. The accuracy of ΔV_p recorded by the syringe pump is ± 0.6 mm^3, the accuracy of the pressure increment measured by the transducer is ± 0.1 MPa, and accuracy of pore volume is ± 130 mm^3. Using this information and considering a pressure increment of $\Delta p = \Delta P = 10.0$ MPa, $K_s'' = 22.0 \pm 0.7$ GPa for Berea sandstone. The accuracy of K_s'' strongly depends on the precision of ΔV_p. Less accurate controller was used for testing shale and the volume of the tubes was large for the low-pressure system, hence $K_s'' = 7 \pm 2$ GPa is reported with less precision for Opalinus clay.

DISCUSSION AND CONCLUSIONS

Indirect estimation of K_s'' from other poroelastic parameters has been reported (Hart and Wang 2010). The generalized Gassmann equation often used in this approach to find K_s'' is

$$K_s'' = \frac{1}{\dfrac{1}{K_f} - \dfrac{\alpha}{K\phi_{int}}\left(\dfrac{1}{B} - 1\right)} \quad (6)$$

where Skempton's coefficient B, drained bulk modulus K, interconnected porosity ϕ_{int}, pore fluid modulus K_f, and Biot coefficient α could be all measured (Detournay and Cheng 1993). Calculated K_s'' value from this approach is very sensitive to the B value and any small uncertainty in the input Skempton's coefficient can drastically change the estimated K_s''. For this reason, many previous works have shown that using this indirect approach provides K_s'' being significantly smaller than K_s' for porous rock (Hart and Wang 2010, Makhnenko and Labuz 2016).

Green and Wang (1986) reported that for Berea sandstone K_s'' approaches K_f at near zero effective stress and, as the Terzaghi effective stress increases, it comes closer to the unjacketed bulk modulus of the rock, *i.e.* $K_f < K_s'' \ll K_s'$. Berge *et al*. (1993) attempted to estimate K_s'' indirectly through other poroelastic measurements and reached the same conclusion. It was suggested that only the direct measurement of this parameter will examine the validity of the assumption $K_s = K_s' = K_s''$. Berge (1998) discussed the dependence of K_s'' on the effective stress suggesting that it is different at low and high effective stresses. However, it was not realized that the estimated values of K_s'' are very sensitive to Skempton's B coefficient. Our direct measurement of K_s'' shows that this parameter, similar to K_s', is independent of Terzaghi effective stress for the sandstone. It is believed that any difference between K_s' and K_s'' is a result of deviation from the ideal porous material assumptions. In the case of Berea sandstone, non-connected porosity (up to 2% of total specimen volume) was detected and caused $K_s'' < K_s' < K_{quartz}$. Results for Opalinus clay provide $K_s'' \approx K_s'$ indicating that, despite of multi-mineral composition of the shale, over a certain range of applied stresses this rock can be considered as an ideal porous material with very small non-connected pore space.

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