Unjacketed bulk compressibility of sandstone in laboratory experiments

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ABSTRACT

The unjacketed bulk modulus of sandstone, K_s', is often considered to be equal to the bulk modulus of quartz, the main mineral which forms it. However, preliminary tests and some other works show that for Berea sandstone this assumption might be violated. Three different types of laboratory experiments were performed on the rock to measure K_s' and Biot's coefficient α: unjacketed plane strain compression, drained compression with water collection, and jacketed/unjacketed hydrostatic compression. The values of these parameters are reported at 5 *MPa* mean Terzaghi effective stress; α is found to be in the range of 0.64 – 0.74 and K_s' = 27.2 – 30.9 *GPa*. The latter value is significantly smaller than the measured bulk modulus of quartz, K_{quartz} = 37.0 *GPa*. It can be explained by the presence of clays or the existence of non-connected pore space in the rock.

INTRODUCTION

Gassmann's equation (Gassmann, 1951) provides a relationship between the elastic constants of a porous material measured under drained (long-time response) and undrained (short-time response) conditions in terms of the material's porosity, compressibility of the solid grains, and compressibility of the pore fluid. The latter affects the overall compressibility and related seismic velocities of rock, and is used in petroleum industry to differentiate between pore fluids. Also, Gassman's equation is used to estimate undrained poroelastic parameters for problems where coupling of external stress and pore pressure is involved (Han and Batzle, 2004; Hart and Wang, 2010).

Gassmann's original analysis assumes homogeneity and isotropy, which leads to the conclusion that the unjacketed bulk modulus K_s' is equal to dominant mineral bulk modulus K_s. Gassman's equation was then generalized by Brown and Korringa (1975) and can be written as

$$K_u = K + \frac{\alpha^2 K}{(1-\alpha)\alpha + \phi K\left(\frac{1}{K_f} - \frac{1}{K_s''}\right)} \quad (1)$$

where ϕ is the porosity of the rock, K and K_u are respectively its drained and undrained bulk moduli, K_f is the bulk modulus of the pore fluid, K_s'' is the unjacketed pore bulk modulus, and α is the effective stress coefficient introduced by Biot (1941):

$$\alpha = 1 - \frac{K}{K_s'} \quad (2)$$

Biot coefficient α should be between 0 and 1 and it expresses the dependence of the mean stress $P = (\sigma_1 + \sigma_2 + \sigma_3)/3$, from the effective mean stress P' and pore pressure p (sign agreement is compression positive):

$$P = P' + \alpha p \quad (3)$$

If $\alpha = 1$, which is the case for some soils where $K \ll K_s'$, then equation (3) becomes an expression for Terzaghi's (1936) effective stress.

Generally, K_s, K_s', and K_s'' do not have to be equal to each other. However, most of the classical poroelastic studies (*e.g.* Rice and Cleary, 1976; Zimmerman *et al.* 1986; Detournay and Cheng, 1993) have used the assumption of their equality. A few experimental works (*e.g.* Hart and Wang, 2010) have shown that this condition is not realized for some rock.

We conducted drained and undrained compression experiments (Makhnenko and Labuz, 2012) with the University of Minnesota (UMN) plane strain apparatus (Labuz *et al.*, 1996). The following values were obtained for K, K_u, and Skempton's coefficient B (corrected for the system response in measuring pore pressure) determined at 5 *MPa* Terzaghi effective mean stress: K = 9.6 *GPa*, K_u = 13.8 *GPa*, and B = 0.54. The following equation gives the relationship between K, K_u, α, and B (Detournay and Cheng, 1993):

$$B = \frac{K_u - K}{\alpha K_u} \quad (4)$$

It can be used to calculate α and K_s' from equations (2) and (4), and the following values were obtained: $\alpha = 0.67$ and $K_s' = 28.9$ *GPa*.

In summary, appropriate determination of unjacketed bulk modulus of the rock is important for calculation of effective stress with respect to poroelastic response. Moreover, very often calculation of some other poroelastic parameters (*e.g.* K_u and K_f in equation (1)) is based on the knowledge of K_s' or α. However, properties of unjacketed bulk modulus are often poorly understood and oversimplified, and only a few measurements of K_s' have been made for sandstones (Han and Batzle, 2004). Therefore, three different types of laboratory experiments were conducted on Berea sandstone to obtain independent measurements of K_s' and α and determine which assumptions are valid.

EXPERIMENTAL METHODS

The UMN Plane-Strain Apparatus allows application of three different principal stresses and measurement of principal strains of prismatic specimens (100 x 87 x 44 *mm*), as well as the ability to apply and measure pore pressure within the rock. Before conducting deviatoric loading on the specimen under drained (pore pressure p is equal to a constant p_o), undrained (the mass of the pore fluid in the rock does not change), or unjacketed (the increase in the mean stress P is equal to the increase in pore pressure p) conditions, Skempton's coefficient B is measured. Full water saturation of the rock is assured by obtaining constant values of B with increasing pore pressure given that the Terzaghi effective stress stays the same (Makhnenko and Labuz, 2013).

Drained compression with water collection (DCW). Performing plane strain tests under drained conditions with $p = 0$ and assuming full saturation of the rock, such that the volume of the fluid in the pores is equal to the pore volume of the specimen, the volume of the fluid drained away from it during elastic deformation can be precisely measured. Thus, it is possible to calculate α as a ratio between the drained fluid volume ΔV_f and the change in specimen volume ΔV (Detournay and Cheng, 1993):

$$\alpha = \frac{\Delta V_f}{\Delta V} \tag{5}$$

Measurements of K then allow the calculation of the unjacketed bulk modulus K_s'.

Unjacketed plane strain compression (UJPSC). The plane strain apparatus can be also used to perform an unjacketed test, characterized by the increase in the mean stress P being equal to the increase in pore pressure p. To achieve this condition, $\sigma_1 = \sigma_2 = \sigma$ can be applied which gives $\sigma_3 = \nu'(\sigma_1 + \sigma_2) = 2\nu'\sigma$ (Makhnenko and Labuz, 2013), where ν' is the unjacketed Poisson's ratio of the material, and it can be measured independently in plane strain compression. Then, the pore pressure can be increased by the same amount as P: $\Delta P = \Delta p = (2 + 2\nu')\sigma/3$, and the unjacketed bulk modulus K_s' can be calculated as

$$K_s' = V \frac{\Delta P}{\Delta V}\bigg|_{\Delta P = \Delta p} \tag{6}$$

where V is the specimen volume.

Jacketed and unjacketed hydrostatic compression (JHC and UHC). Additionally, jacketed and unjacketed hydrostatic compression tests were performed on prismatic sandstone specimens instrumented with three sets of strain rosettes (Figure 1).

Figure 1: Specimen for jacketed compression test.

Dry sandstone specimens covered with polyurethane were placed inside a pressure chamber and loaded to 60 MPa hydrostatic pressure. Measured jacketed bulk modulus K is

$$K = V\frac{\Delta P}{\Delta V}\bigg|_{jacketed} \tag{7}$$

It can be considered as the drained bulk modulus of the material, because in the dry test pore pressure in the material does not change ($p = 0$).

After that, the jacket is removed and the confining fluid (hydraulic oil), which does not have a chemical effect on Berea sandstone in the short term, is allowed to penetrate into the rock. The unjacketed bulk modulus is calculated from

$$K_s' = V\frac{\Delta P}{\Delta V}\bigg|_{unjacketed} \tag{8}$$

Summary of the experimental methods. The description of the performed tests (abbreviations of test names are used) in terms of boundary conditions and measured and calculated parameters is presented in Table 1.

Table 1. Summary of poroelasticity tests.

Test	Boundary conditions	Measurements	Parameters
DCW	$p = 0$	ΔP, ΔV, ΔV_f	$K = V\Delta P/\Delta V$, $\alpha = \Delta V_f/\Delta V$
UJPSC	$\Delta P = \Delta p$	ΔP, ΔV, Δp	$K_s' = V\Delta P/\Delta V$
JHC	*dry:* $p = 0$	ΔP, ΔV	$K = V\Delta P/\Delta V$
UHC	$P = p$, $\Delta P = \Delta p$	ΔP, ΔV	$K_s' = V\Delta P/\Delta V$

EXPERIMENTAL RESULTS

Plane strain and hydrostatic compression tests were performed on liquid-saturated Berea sandstone specimens, which were cut from the same block at the same orientation. The tested material exhibited slight (5%) anisotropy in P-wave velocity, with a porosity $\phi = 0.23$ and permeability $k = 40$ *mD* (measured at 5 *MPa* confinement).

Drained compression with water collection (DCW). The bulk modulus measured in the plane strain compression under drained conditions ($P = 5$ *MPa*, $p = 0$ *MPa*) is found to be in the range $K = 9.4 – 9.8$ *GPa*. Around 0.20 *ml* of water came out of the specimen (typical specimen volume $V = 380$ *ml*) in each of the three tests of this type during elastic loading. Biot's modulus α is calculated to be in the range $\alpha = 0.64$ – 0.71. This measurement is not precise because the water drained from the specimen came in the form of small drops 0.02 – 0.03 *ml* each.

Unjacketed plane strain compression (UJPSC). The unjacketed Poisson's ratio of Berea sandstone ν' was measured to be equal to 0.28. Then, three unjacketed tests were performed at $p = 4 – 5$ *MPa* and $P = 9 – 10$ *MPa* and the unjacketed bulk modulus is determined to be in the range $K_s' = 29.1 – 30.4$ *GPa*.

Jacketed and unjacketed hydrostatic compression (JHC and UHC). Three prismatic 53 x 35 x 35 *mm* specimens were tested under jacketed and then unjacketed conditions. Linear strains measured in the direction perpendicular to bedding planes are 5 – 8% smaller than those along the beds, which confirms a slight material anisotropy. Bulk moduli calculated from jacketed hydrostatic compression tests are found to be increasing with pressure: $K = 5$ *GPa* for $P = 2$ *MPa*, $K = 8$ *GPa* at $P = 5$ – 6 *MPa* , and $K = 13$ *GPa* (and constant) when hydrostatic pressure exceeded 30 MPa. The unjacketed bulk modulus is measured to be constant for pressures from 1 – 60 MPa and equal to $K_s' = 29.4$ - 30.9 *GPa*. Additionally, a fused quartz specimen was tested; its volumetric strain response is linear up to $P = 60$ *MPa* and its bulk modulus is $K_{quartz} = 37.0$ *GPa*. The results of the hydrostatic compression experiments are presented in Figure 2.

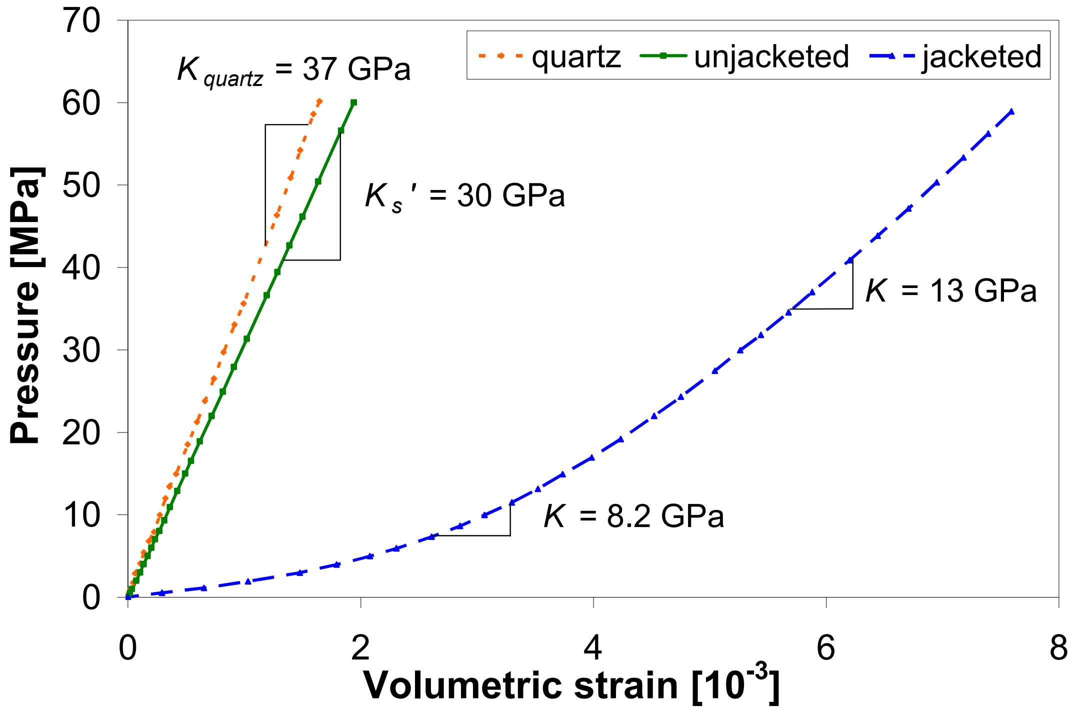

Figure 2: Results of jacketed and unjacketed compression tests.

Results summary. Measured and calculated bulk moduli K and K_s', along with the Biot coefficient α, are presented in Table 2. All the values are reported for the Terzaghi effective mean stress equal to 5 *MPa*.

Table 2. Experimental results.

Test	K [*GPa*]	K_s' [*GPa*]	α
DCW	9.4 - 9.8	27.2 - 32.4	0.64 - 0.71
UJPS	*9.4 - 9.8	29.1 - 30.4	0.66 - 0.69
JHC and UHC	7.9 - 8.4	29.4 - 30.9	0.71 - 0.74

* - value of K is taken from DCW test

DISCUSSION AND CONCLUSIONS

The three proposed testing methods provided values of α = 0.64 – 0.74 and K_s' in the range of 27.2 – 30.9 *GPa* measured at 5 *MPa* effective (Terzaghi's) mean stress. Therefore, it can be stated that the unjacketed bulk modulus of Berea sandstone is smaller than the quartz grain bulk modulus (37.0 *GPa*).

The difference between unjacketed bulk modulus and bulk modulus of quartz – the main mineral which forms the rock, can be explained by the fact that two of Gassmann's assumptions are violated, namely monomineralic rock and well connected pore space. Petrographic analysis of Berea sandstone (*e.g.* Cristensen and

Wang, 1996; Hart and Wang, 2010) shows the presence of the clay cement between quartz grains in the rock, which can increase its unjacketed compressibility. Some other studies (Dullien, 1992) report the existence of non-connected pore space in Berea sandstone, which also increases its unjacketed compressibility (Dormieux *et al.*, 2002).

Calculation of some poroelastic parameters in the field is based on knowledge of the compressibility of the rock matrix in the generalized Gassman's equation (eqn. 2). It is shown that in some cases taking it as the compressibility of the dominant mineral in the rock can lead to its overestimation up to 25% and thus make the prediction of some water-saturation effects substantially wrong.

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